From Table I we find $p = 3.013$ at $-5°$, which, increased by 0.075%, gives 3.015, the vapor pressure of ice in contact with the atmosphere at $-5°$ C.

### THE VAPOR PRESSURE OF WATER IN EQUILIBRIUM WITH THE ATMOSPHERE

Just as in the case of ice, the vapor pressure of water in contact with the atmosphere differs from its vapor pressure when in contact with its own vapor only. For water below 0° C. the correction may, for all practical purposes, be computed from equation (14). For higher temperatures, the following more exact equations may be employed:

$$100\frac{\Delta p}{p} = 0.0775 - 3.13 \times 10^{-4}\, t \text{ (valid up to } t = 40° \text{ C.)} \quad (15)$$

and

$$100\frac{\Delta p}{p} = 0.0652 - 8.75 \times 10^{-5}\, p \text{ (valid above } 50° \text{ C.)} \quad (16)$$

In these equations both the effect of the external pressure, as such, and the effect of dissolved air are taken into account. At high temperatures (70°–80° C.) this correction amounts to more than 0.1 mm.

TABLE 1.—*The vapor pressure of ice*

Computed from the equation

$$\log_{10} p = \frac{-2445.5646}{T} + 8.2312 \log_{10} T - 0.01677006\, T + 1.20514 \times 10^{-5}\, T^2 - 6.757169\, T \text{ mm. Hg.}$$

Based upon the measurements of Weber (Comm. Phys. Lab. Leyden No. 150, 37 1915]) and Scheel and Heuse (Ann. Physik 29, 731 [1909]).

$T = 273.1 + t$

−90° to −30°. unit, 0.001 mm. Hg

| t | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| ° C. | | | | | | | | | | |
| −90 | $0.07_0$ | $0.05_8$ | $0.04_8$ | $0.04_0$ | $0.03_3$ | $0.02_7$ | $0.02_2$ | $0.01_8$ | $0.01_5$ | $0.01_2$ |
| −80 | 0.40 | 0.34 | 0.29 | 0.24 | 0.20 | 0.17 | 0.14 | 0.12 | 0.10 | $0.08_4$ |
| −70 | 1.94 | 1.67 | 1.43 | 1.23 | 1.05 | 0.90 | 0.77 | 0.66 | 0.56 | 0.47 |
| −60 | $8.0_9$ | $7.0_3$ | $6.1_4$ | $5.3_4$ | $4.6_4$ | $4.0_3$ | $3.4_9$ | $3.0_2$ | $2.6_1$ | $2.2_5$ |
| −50 | $29.5_5$ | 26.1 | 23.0 | 20.3 | 17.8 | 15.7 | 13.8 | 12.1 | 10.6 | $09.2_5$ |
| −40 | 96.6 | 86.2 | 76.8 | 68.4 | 60.9 | 54.1 | 48.1 | 42.6 | 37.8 | 33.4 |
| −30 | 285.9 | 257.5 | 231.8 | 208.4 | 187.3 | 168.1 | 150.7 | 135.1 | 120.9 | 108.1 |

−30° to 0°. mm. Hg

| t | 0.0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
|---|---|---|---|---|---|---|---|---|---|---|
| ° C. | | | | | | | | | | |
| −29 | 0.317 | 0.314 | 0.311 | 0.307 | 0.304 | 0.301 | 0.298 | 0.295 | 0.292 | 0.289 |
| −28 | 0.351 | 0.348 | 0.344 | 0.341 | 0.337 | 0.334 | 0.330 | 0.327 | 0.324 | 0.320 |
| −27 | 0.389 | 0.385 | 0.381 | 0.377 | 0.374 | 0.370 | 0.366 | 0.362 | 0.359 | 0.355 |
| −26 | 0.430 | 0.426 | 0.422 | 0.418 | 0.414 | 0.409 | 0.405 | 0.401 | 0.397 | 0.393 |
| −25 | 0.476 | 0.471 | 0.467 | 0.462 | 0.457 | 0.453 | 0.448 | 0.444 | 0.439 | 0.435 |
| −24 | 0.526 | 0.520 | 0.515 | 0.510 | 0.505 | 0.500 | 0.495 | 0.490 | 0.486 | 0.481 |
| −23 | 0.580 | 0.574 | 0.569 | 0.563 | 0.558 | 0.552 | 0.547 | 0.541 | 0.536 | 0.531 |
| −22 | 0.640 | 0.633 | 0.627 | 0.621 | 0.615 | 0.609 | 0.603 | 0.597 | 0.592 | 0.586 |
| −21 | 0.705 | 0.698 | 0.691 | 0.685 | 0.678 | 0.672 | 0.665 | 0.659 | 0.652 | 0.646 |
| −20 | 0.776 | 0.769 | 0.761 | 0.754 | 0.747 | 0.740 | 0.733 | 0.726 | 0.719 | 0.712 |
| −19 | 0.854 | 0.846 | 0.838 | 0.830 | 0.822 | 0.814 | 0.806 | 0.799 | 0.791 | 0.783 |
| −18 | 0.939 | 0.930 | 0.921 | 0.912 | 0.904 | 0.895 | 0.887 | 0.879 | 0.870 | 0.862 |
| −17 | 1.031 | 1.021 | 1.012 | 1.002 | 0.993 | 0.984 | 0.975 | 0.966 | 0.956 | 0.947 |
| −16 | 1.132 | 1.121 | 1.111 | 1.101 | 1.091 | 1.080 | 1.070 | 1.060 | 1.051 | 1.041 |
| −15 | 1.241 | 1.230 | 1.219 | 1.208 | 1.196 | 1.186 | 1.175 | 1.164 | 1.153 | 1.142 |
| −14 | 1.361 | 1.348 | 1.336 | 1.324 | 1.312 | 1.300 | 1.288 | 1.276 | 1.264 | 1.253 |
| −13 | 1.490 | 1.477 | 1.464 | 1.450 | 1.437 | 1.424 | 1.411 | 1.399 | 1.386 | 1.373 |
| −12 | 1.632 | 1.617 | 1.602 | 1.588 | 1.574 | 1.559 | 1.546 | 1.532 | 1.518 | 1.504 |
| −11 | 1.785 | 1.769 | 1.753 | 1.737 | 1.722 | 1.707 | 1.691 | 1.676 | 1.661 | 1.646 |
| −10 | 1.950 | 1.934 | 1.916 | 1.899 | 1.883 | 1.866 | 1.849 | 1.833 | 1.817 | 1.800 |
| −9 | 2.131 | 2.112 | 2.093 | 2.075 | 2.057 | 2.039 | 2.021 | 2.003 | 1.985 | 1.968 |
| −8 | 2.326 | 2.306 | 2.285 | 2.266 | 2.246 | 2.226 | 2.207 | 2.187 | 2.168 | 2.149 |
| −7 | 2.537 | 2.515 | 2.493 | 2.472 | 2.450 | 2.429 | 2.408 | 2.387 | 2.367 | 2.346 |
| −6 | 2.765 | 2.742 | 2.718 | 2.695 | 2.672 | 2.649 | 2.626 | 2.603 | 2.581 | 2.559 |
| −5 | 3.013 | 2.987 | 2.962 | 2.937 | 2.912 | 2.887 | 2.862 | 2.838 | 2.813 | 2.790 |
| −4 | 3.280 | 3.252 | 3.225 | 3.198 | 3.171 | 3.144 | 3.117 | 3.091 | 3.065 | 3.039 |
| −3 | 3.568 | 3.539 | 3.509 | 3.480 | 3.451 | 3.422 | 3.393 | 3.364 | 3.336 | 3.308 |
| −2 | 3.880 | 3.848 | 3.816 | 3.785 | 3.753 | 3.722 | 3.691 | 3.660 | 3.630 | 3.599 |
| −1 | 4.217 | 4.182 | 4.147 | 4.113 | 4.079 | 4.045 | 4.012 | 3.979 | 3.946 | 3.913 |
| −0 | 4.579 | 4.542 | 4.504 | 4.467 | 4.431 | 4.395 | 4.359 | 4.323 | 4.287 | 4.252 |

TABLE 2.—*The vapor pressure of liquid water from −16° C. to 0° C. (in mm. Hg)*

Computed from Table 1 with the aid of the thermodynamic equation

$$\log_{10}\frac{p_w}{p_i} = \frac{-1.1489\, t}{273.1 + t} - 1.330 \times 10^{-5} t^2 + 9.084 \times 10^{-8} t^3$$

| t | 0.0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
|---|---|---|---|---|---|---|---|---|---|---|
| ° C. | | | | | | | | | | |
| −15 | 1.436 | 1.425 | 1.414 | 1.402 | 1.390 | 1.379 | 1.368 | 1.356 | 1.345 | 1.334 |
| −14 | 1.560 | 1.547 | 1.534 | 1.522 | 1.511 | 1.497 | 1.485 | 1.472 | 1.460 | 1.449 |
| −13 | 1.691 | 1.678 | 1.665 | 1.651 | 1.637 | 1.624 | 1.611 | 1.599 | 1.585 | 1.572 |
| −12 | 1.834 | 1.819 | 1.804 | 1.790 | 1.776 | 1.761 | 1.748 | 1.734 | 1.720 | 1.705 |
| −11 | 1.987 | 1.971 | 1.955 | 1.939 | 1.924 | 1.909 | 1.893 | 1.878 | 1.863 | 1.848 |
| −10 | 2.149 | 2.134 | 2.116 | 2.099 | 2.084 | 2.067 | 2.050 | 2.034 | 2.018 | 2.001 |
| −9 | 2.326 | 2.307 | 2.289 | 2.271 | 2.254 | 2.236 | 2.219 | 2.201 | 2.184 | 2.167 |
| −8 | 2.514 | 2.495 | 2.475 | 2.456 | 2.437 | 2.418 | 2.399 | 2.380 | 2.362 | 2.343 |
| −7 | 2.715 | 2.695 | 2.674 | 2.654 | 2.633 | 2.613 | 2.593 | 2.572 | 2.553 | 2.533 |
| −6 | 2.931 | 2.909 | 2.887 | 2.866 | 2.843 | 2.822 | 2.800 | 2.778 | 2.757 | 2.736 |
| −5 | 3.163 | 3.139 | 3.115 | 3.092 | 3.069 | 3.046 | 3.022 | 3.000 | 2.976 | 2.955 |
| −4 | 3.410 | 3.384 | 3.359 | 3.334 | 3.309 | 3.284 | 3.259 | 3.235 | 3.211 | 3.187 |
| −3 | 3.673 | 3.647 | 3.620 | 3.593 | 3.567 | 3.540 | 3.514 | 3.487 | 3.461 | 3.436 |
| −2 | 3.956 | 3.927 | 3.898 | 3.871 | 3.841 | 3.813 | 3.785 | 3.757 | 3.730 | 3.702 |
| −1 | 4.258 | 4.227 | 4.196 | 4.165 | 4.135 | 4.105 | 4.075 | 4.045 | 4.016 | 3.986 |
| −0 | 4.579 | 4.546 | 4.513 | 4.480 | 4.448 | 4.416 | 4.385 | 4.353 | 4.320 | 4.289 |

## TEMPERATURE LAG OF THE OCEANS

551.463

By W. J. HUMPHREYS

[Weather Bureau, Washington, D. C.]

From time to time we see the assertion that any appreciable change in the temperature of the ocean, such, for instance, as might result from a change in the solar constant, persists, in large measure, over a period of years. These assertions, however, are not backed up by sound theory, but based, if upon anything at all, upon a few observations, which, for the particular long-range forecast, or other objects in view, it is convenient to assume explained in the alleged manner.

It may be helpful, therefore, to compute, according to known physical laws and constants, how great we reasonably may expect such lags to be.

Owing to wave action and convection, temperature changes of the ocean are not confined to the surface, but penetrate in approximately full magnitude to an appreciable depth. For calculation it is here tentatively assumed that an exactly equal change extends down 20 meters, and that beyond this depth there is no change. It is believed that this assumption is of the proper order. Anyhow, the results will be correct to within an experimental factor.

As is well known, the earth maintains its average balance between heat gained and heat lost, by radiating, in amount, as would a black body of the same size at the absolute temperature 260°.[1] It is also known that, on the average, temperature changes are roughly the same at the surface and all the way up through the troposphere, or convective portion of the atmosphere. Hence, if the surface of the earth should be kept at a slightly different temperature than heretofore, its planetary black-body temperature would also be altered in the same sense and to about the same extent.

The surface temperature of the land has a shorter time lag than that of the ocean, owing to permanency of position, lack of convection, and the small, as compared with water, specific heat of soil and rock. Actually, therefore, the temperature lags of the earth as a whole are less than they would be if there were no land at all, but for islands and many coastal regions not greatly less. Hence, to obtain the upper limit solution of the problem this land effect will be omitted. Perhaps this effect can best be

[1] Abbot and Fowle, Annals Astrophys. Obs. Smithsonian Inst., 2. p. 175.

(To face p. 491)

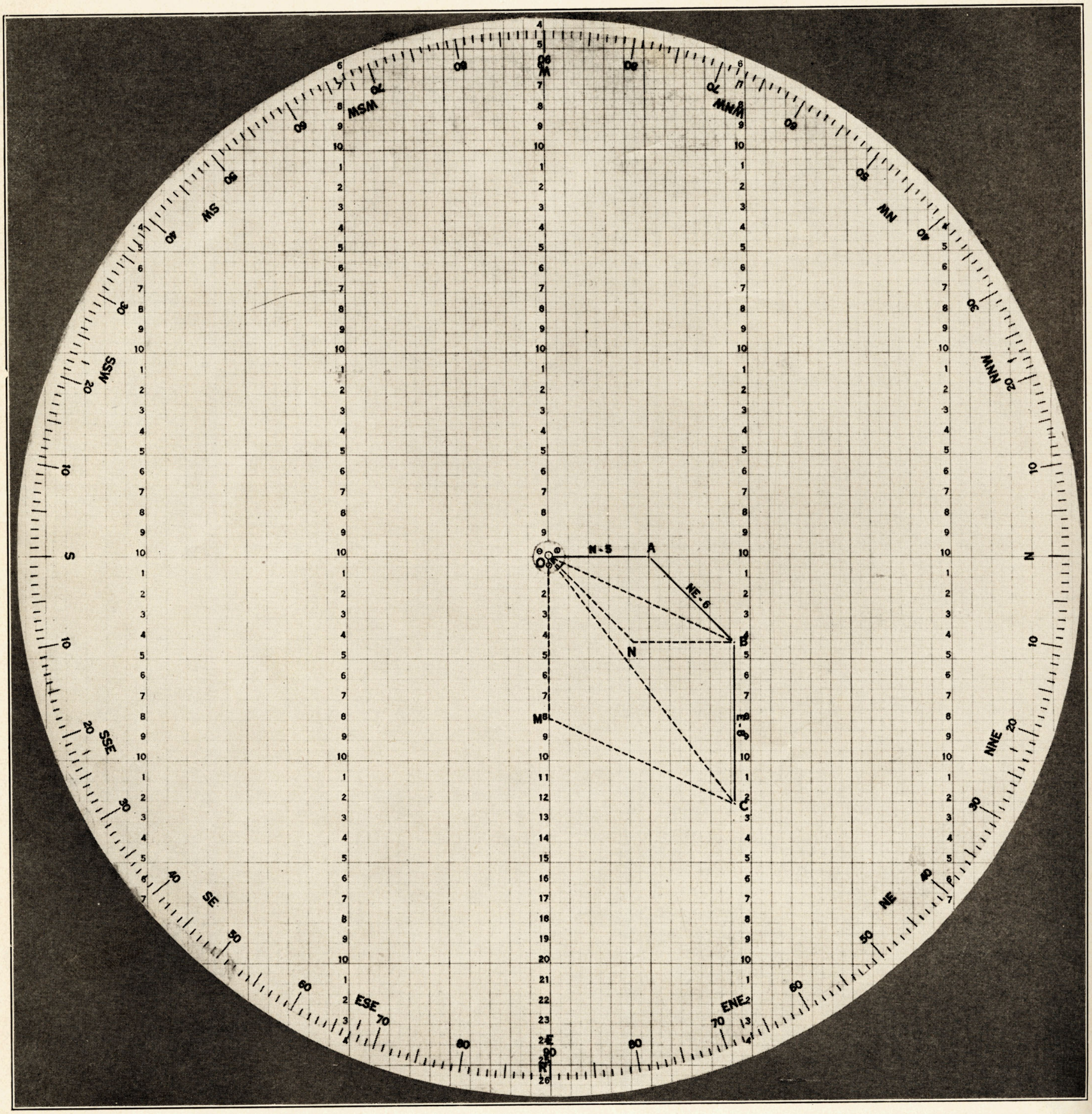


FIG. 1.—Plotting board for compounding vector quantities

treated as a correction term of one or another value as circumstances may determine.

Let, then, the oceans become warmer (if cooler the method of calculation would be the same) than is their wont by 1° C. from the surface to the depth of 20 meters, a storage of 2,000, approximately, calories per square centimeter of surface, and let the previous temperatures remain unchanged below this level. In what times, under normal conditions, will half, three-fourths, and nine-tenths of this added heat be lost and the excess temperature have fallen to 0.5° C., 0.25° C., and 0.1° C. respectively?

Since the loss of heat by radiation per square centimeter of flat surface, or its equivalent, of a black body is $1.27 \times 10^{-12}\theta^4$ calories per second, in which $\theta$ is the absolute temperature, it follows that the net rate, calories per second, at which the stored up heat $Q$, is lost per square centimeter of ocean surface is the difference between the rates of total loss and total gain; that is, in symbols

$$dQ = \frac{1.27}{10^{12}}\left\{\theta^4 - (260)^4\right\}dt,$$

in which $t$ is the time in seconds.

If $m$ is the mass of water warmed, $s$ its specific heat, then

$$dQ = msd\theta$$

but in this case $m = 2{,}000$ and $s = 1$. Hence, substituting

$$\frac{d\theta}{\theta^4 - (260)^4} = \frac{1.27}{2 \times 10^{15}}\,dt$$

If, as assumed, the initial planetary temperature is 261° absolute, then the time in seconds for the given mass of water to cool to $\theta_1$ is found by the equation

$$\int_{\theta_1}^{261} \frac{d\theta}{\theta^4 - (260)^4} = \frac{1.27}{2 \times 10^{15}} \int_0^t dt$$

or

$$\left[\log_e \left(\frac{\theta - 260}{\theta + 260}\right)^{\frac{1}{4(260)^3}}\right]_{\theta_1}^{261} - \frac{1}{2(260)^3}\left[\tan^{-1}\frac{\theta}{260}\right]_{\theta_1}^{261} = \frac{1.27}{2 \times 10^{15}}\,t$$

in which $t$, as explained, is seconds, and the angle in radians.

The required times—time to lose, under normal conditions, half, three-fourths, and nine-tenths of the accumulated heat—are found by substituting for $\theta_1$ in this last equation 260.5, 260.25, and 260.1, respectively. The results are

| | | | |
|---|---|---|---|
| $\theta_1 =$ | 260. 5° | 260. 25° | 260. 1° |
| $t$, days = | 178. 958 | 358. 287 | 595. 625 |

Obviously, then, the temperature lags of the ocean incident to variations of incoming radiation are not very great, save for small and comparatively ineffective residuals.

## GRAPHICAL METHOD OF COMPOUNDING VECTORS

551.501

By WILLIAM C. HAINES

[Weather Bureau, Washington, D. C., August, 1924]

The composition of vector quantities may be accomplished by either one of two methods; namely, the analytical method or the graphical method.

The analytical method of compounding vector quantities is expressed by the formulae:

$$\tan\theta = \frac{Y}{X} = \frac{r_1 \sin\alpha_1, + r_2 \sin\alpha_2 + r_3 \sin\alpha_3 + \ . \ . \ .}{r_1 \cos\alpha_1, + r_2 \cos\alpha_2 + r_3 \cos\alpha_3 + \ . \ . \ .}$$

$$R = \sqrt{X^2 + Y^2}$$

where $r_1, r_2, r_3$ . . . are vectors, $\alpha_1, \alpha_2, \alpha_3$, . . . are, respectively, the angles which the vectors make with the $X$ axis and $\theta$ is the angle the resultant, $R$, makes with $X$ axis. $X$ and $Y$ are $\Sigma r \cos\alpha$ and $\Sigma r \sin\alpha$, respectively. The composition of vectors by this general method is both tedious and laborious, but this is the proper method to use where mathematical accuracy is required.

The graphical method is accomplished by forming a parallelogram with two of the vectors as sides. The diagonal of this parallelogram and the third vector is taken as the sides of a second parallelogram, etc. The diagonal of the last parallelogram formed is the resultant. The accuracy of the result of this method depends entirely upon the precision with which the vectors are laid off. The writer has devised a simple graphical method by which vectors may be compounded quickly and with surprising accuracy. It is accomplished by means of a so-called plotting board somewhat similar to the one used in connection with our pilot balloon work, except that it is smaller in size.

The plotting board consists of an ordinary drawing board, over the central portion of which is glued a circular sheet of millimeter cross-section paper. Upon this board is mounted a circular celluloid protractor, fastened by a brass bearing at the center. The protractor is graduated in degrees; also, directions to 16 points of the compass are indicated to facilitate the compounding of wind vectors. The celluloid disc should be 50 or 60 cm. in diameter and frosted on one side so that it will take pencil marks readily and still be transparent. An initial line OR (see fig. 1) and scale are marked on the cross-section paper base. Other supplementary scales are indicated, the advantage of which will be explained later.

The use of this graphical method is made clear by the following simple example: Suppose we are to find the resultant direction and velocity of the vectors N–5, NE–6, and E–8.

First step: Set the north point of the protractor on the initial line OR, and with a pencil mark the point A on the protractor at a distance of 5 from the center, as shown by the scale along the line OR. Next turn the protractor to set NE on the line OR, then take the length of the second vector, or 6 from the point A parallel to the line OR, marking the point B on the protractor as the end point of the second vector. In reality a parallelogram OABN is thus formed with the N–5 and NE–6 vectors as sides; therefore the diagonal OB is the